

# Reconstruction of Ocean Currents From Sparse and Noisy Data

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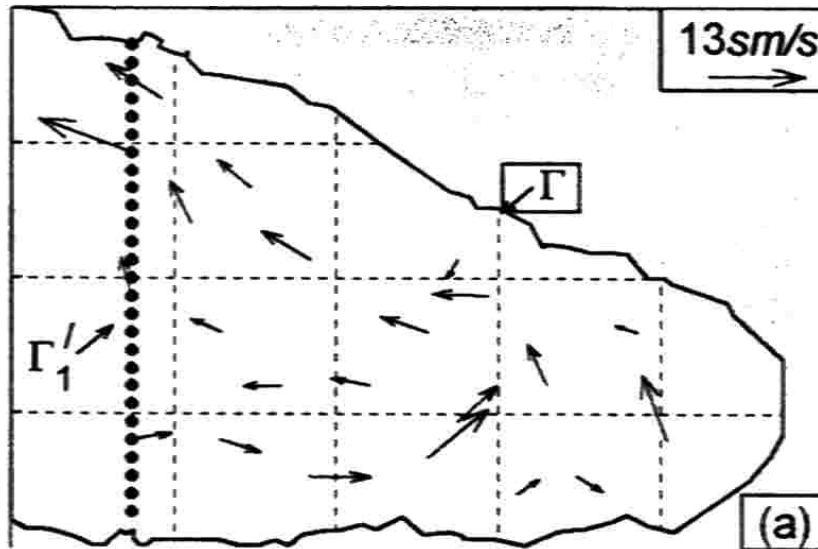
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# Can we get the velocity signal from sparse and noisy data?

- Black Sea



- How can we assimilate sparse and noisy velocity data into numerical model?

# Flow Decomposition

- 2 D Flow (Helmholtz)

$$u = - \partial \psi / \partial y + \partial \phi / \partial x$$

$$v = \partial \psi / \partial x + \partial \phi / \partial y$$

- 3D Flow (Toroidal & Poloidal): Very popular in astrophysics

$$\mathbf{u} = \mathbf{r} \times \nabla A_1 + \mathbf{r} A_2 + \nabla A_3$$

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# 3D Incompressible Flow

- If Incompressible  $\nabla \cdot \mathbf{u} = 0$
- We have

$$\mathbf{u} = \nabla \times (\mathbf{r}\Psi) + \nabla \times \nabla \times (\mathbf{r}\Phi).$$

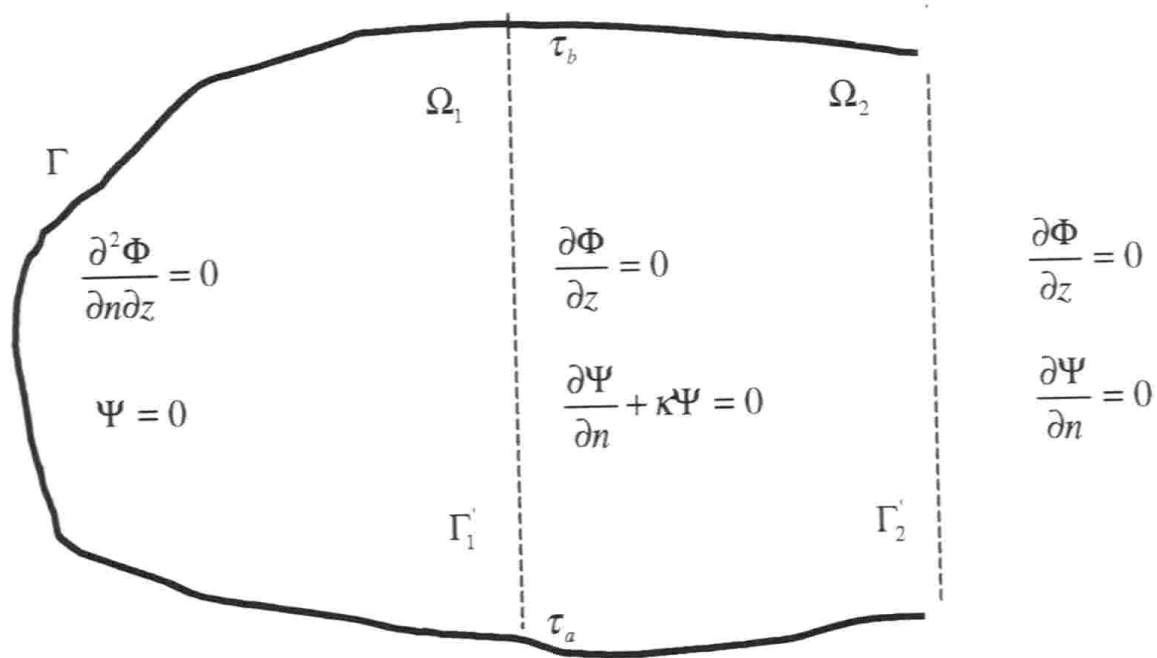
# Flow Decomposition

$$u = \frac{\partial \Psi}{\partial y} + \frac{\partial^2 \Phi}{\partial x \partial z}, \quad v = -\frac{\partial \Psi}{\partial x} + \frac{\partial^2 \Phi}{\partial y \partial z},$$

$$\nabla^2 \Psi = -\zeta, \quad \zeta \text{ is relative vorticity}$$

$$\nabla^2 \Phi = -w$$

# Boundary Conditions



# Basis Functions

$$\Psi(x, y, z, t^\circ) = \sum_{k=1}^{\infty} a_k(z, t^\circ) \Psi_k(x, y, z, \kappa^\circ),$$

$$\frac{\partial \Phi(x, y, z, t^\circ)}{\partial z} = \sum_{m=1}^{\infty} b_m(z, t^\circ) \Phi_m(x, y, z),$$



# Determination of Basis Functions

- Poisson Equations
- $\Gamma$  - Rigid Boundary
- $\Gamma'$  - Open Boundary
- $\{\lambda_k\}$ ,  $\{\mu_m\}$  are Eigenvalues.
- Basis Functions are predetermined.

$$\Delta \Psi_k = -\lambda_k \Psi_k,$$

$$\Delta \Phi_m = -\mu_m \Phi_m,$$

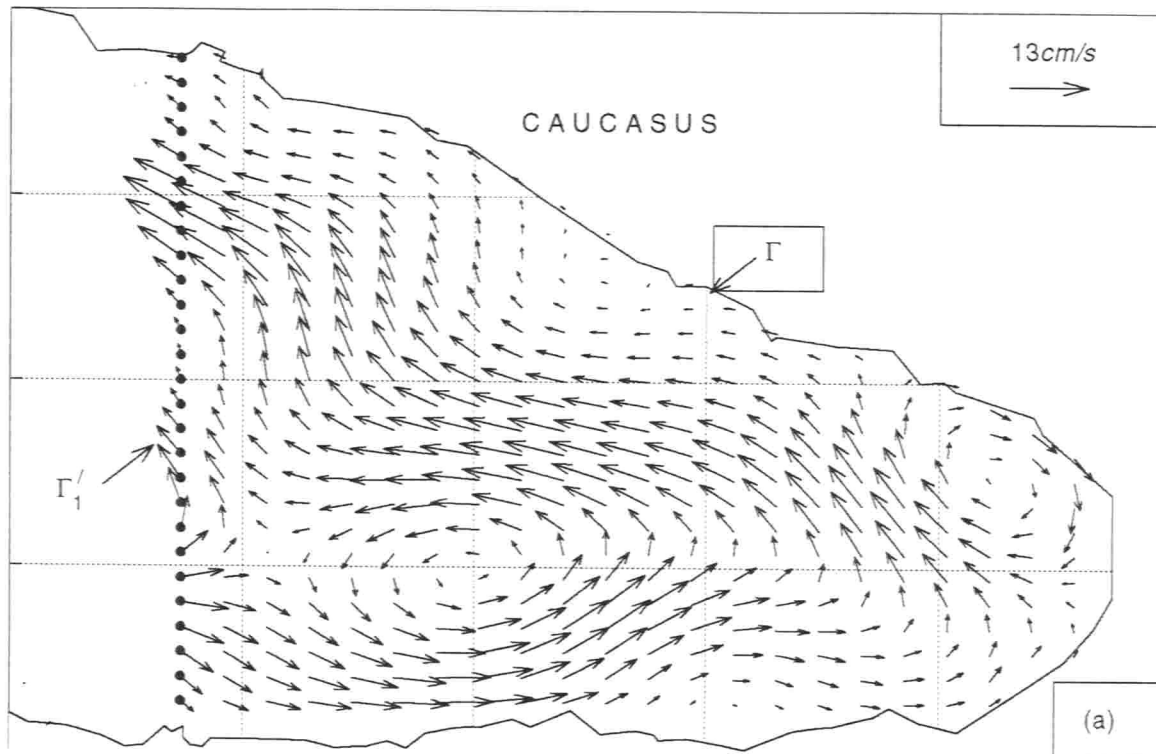
$$\Psi_k|_{\Gamma} = 0, \quad \frac{\partial \Phi_m}{\partial n}|_{\Gamma} = 0,$$

$$\left[ \frac{\partial \Psi_k}{\partial n} + \kappa(\tau) \Psi_k \right] |_{\Gamma'} = 0, \quad \Phi_m|_{\Gamma'} = 0.$$

# *Flow Reconstruction*

$$\begin{aligned}u_{KM} &= \sum_{k=1}^K a_k(z, t^\circ) \frac{\partial \Psi_k(x, y, z, \kappa^\circ)}{\partial y} + \sum_{m=1}^M b_m(z, t^\circ) \frac{\partial \Phi_m(x, y, z)}{\partial x}, \\v_{KM} &= - \sum_{k=1}^K a_k(z, t^\circ) \frac{\partial \Psi_k(x, y, z, \kappa^\circ)}{\partial x} + \sum_{m=1}^M b_m(z, t^\circ) \frac{\partial \Phi_m(x, y, z)}{\partial y}\end{aligned}$$

# Reconstructed Circulation



# Conclusions

- Reconstruction is a useful tool for processing real-time velocity data with short duration and limited-area sampling.
- The scheme can handle highly noisy data.
- The scheme is model independent.
- The scheme can be used for assimilating sparse velocity data